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General error function of synthetic-heterodyne signal processing in interferometric fibre-optic sensors

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A theoretical and experimental investigation, which allows the quantification of the general error function relative to the synthetic-heterodyne signal processing in interferometric fibre-optic sensors, is performed. The implications on the performance of interferometric sensing arrays are addressed. Considering the decrease of the readout phase errors, potentially more favourable implementations of this processing technique are proposed and compared.

1. Introduction

Over the past 15 years fibre sensing technology has undergone considerable progress, helped by important characteristics such as immunity to electromagnetic interference, a non-electrical method of operation, small size and weight, generally low power consumption and, in many cases, comparatively low cost [1,2]. A significant fraction of the research and development effort in this area has been allocated to interferometry-based fibre-optic sensors, due to the potential measurement capabilities they offer. In this context, a number of optical and optoelectronic techniques for demodulation of this type of sensors have been developed [3].

To take full advantage of the optical properties of these sensors it is highly desirable that the demodulation schemes be entirely electrically passive, i.e. not requiring any electric connection to the interferometer and also having no electronic components. Examples of processing schemes with these characteristics are the phase generated carrier [4,5], the ramp-based pseudo-heterodyne [6] and the synthetic-heterodyne [7,8]. All of these require modulation of the interferometric phase with a well defined amplitude, which is obtained by having a non-zero path imbalance for the interferometer combined with an optical frequency modulation with precise amplitude of the light injected into the system. This frequency modulation is normally obtained via modulation of the laser diode injection current. When interrogating single sensors this condition is, in practice, straightforward to fulfil exactly by fine tuning the laser diode injection current modulation amplitude (or modulation depth). However, the situation is different when the objective is to multiplex an array of remote and passive interferometric fibre-optic sensors interrogated by a single optoelectronic unit. In this case, considering that it is hardly feasible to have all the interferometers with exactly the same path-imbalance, the condition for proper demodulation cannot be fulfilled. The consequence is the presence of readout phase errors originating calibration problems and fluctuations of the small signal sensitivity. It is therefore clearly important to study the performance of a given demodulation scheme in the more general situation where the exact working conditions do not occur.

Lewin *et al.* performed an analysis of this type for the pseudo-heterodyne scheme based on the gating of the interferogram [9,10]. In this context, we analyse here the synthetic-heterodyne processing scheme and, considering the decrease of the readout phase errors, potentially more favourable implementations of this technique are proposed and compared.

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2. Theoretical analysis

Modulation of the laser frequency by applying a sinusoidal waveform, $\Delta i_m \sin \omega_m t$, to the injection current of the laser diode, gives rise to an interferometric phase modulation

$\Delta\phi$, at the output of an unbalanced interferometer given by

$$\Delta\phi = \frac{2\pi n\Delta L}{c}\Delta i_m \frac{\delta v}{\delta i}\sin\omega_m t = \Delta\phi_m \sin\omega_m t, \quad (1)$$

where n is the mode effective refractive index, ΔL is the interferometer path imbalance and $\delta v/\delta i$ is the effective current-to-frequency conversion factor of the laser diode. Writing the interferometer phase shift as $\phi(t) + \Delta\phi$, where $\phi(t)$ contains the signal phase information of interest $\phi_s(t)$, and a drift component $\phi_d(t)$, the output after the detection and amplification block can be expressed as

$$V = \alpha I_0\{1 + k\cos[\phi(t) + \Delta\phi_m \sin\omega_m t]\}, \quad (2)$$

where I_0 is the optical input intensity, k is the fringe visibility, and α is a constant factor taking into account the losses in the system and the gain of the detection and amplification block. Expanding equation (2) in terms of first-kind Bessel functions ($J_a(\Delta\phi_m)$; $a = 0, 1, 2, \ldots$), the amplitudes of the components at the fundamental (ω_m) and the second harmonic ($2\omega_m$) of the laser modulation frequency will be

$$S_1 = -\alpha I_0 k J_1(\Delta\phi_m)\sin\phi(t) \quad (3.1)$$

$$S_2 = \alpha I_0 k J_2(\Delta\phi_m)\cos\phi(t). \quad (3.2)$$

Multiplying equation (3.1) by $\sin\omega_c t$ and equation (3.2) by $\cos\omega_c t$ (an electronically generated carrier signal), and adding both, the following relation results

$$S_0 = \alpha I_0 k[J_2(\Delta\phi_m)\cos\phi(t)\cos\omega_c t - J_1(\Delta\phi_m)\sin\phi(t)\sin\omega_c t] \quad (4)$$

If the laser modulation depth is adjusted such that $J_1(\Delta\phi_m) = J_2(\Delta\phi_m)$, i.e. $\Delta\phi_m = 2{\cdot}63$ rad, equation (4) becomes [3]

$$S_0|_{J_1=J_2} = \alpha I_0 k J_1(\Delta\phi_m)\cos[\omega_c t + \phi(t)]. \quad (5)$$

From this equation it is straightforward to recover the phase information $\phi_s(t)$ by using a frequency discriminator or a phase-locked loop (PLL) with subsequent integration. Figure 1 shows a block diagram of this processing scheme. However, in an array of interferometric sensors it is not possible to fulfil the condition $J_1(\Delta\phi_m) = J_2(\Delta\phi_m)$ for all sensors in the network because, in real applications, the path imbalances can hardly be identical (this means that, in general, the laser modulation depth can only be adjusted to a single sensor in the network). The study of this problem indicates that when the condition $J_1(\Delta\phi_m) = J_2(\Delta\phi_m)$ is not fulfilled, equation (5) becomes:

$$S_0|_{J_1\neq J_2} = \alpha I_0 k J_1(\Delta\phi_m)F(\zeta_{21},\phi)\cos[\omega_c t + \psi(\zeta_{21},\phi)] \quad (6)$$

where

$$F(\zeta_{21},\phi) = \left(\frac{1 + \zeta_{21}^2 - (1 - \zeta_{21}^2)\cos(2\phi(t))}{2}\right)^{1/2} \quad (7)$$

$$\psi(\zeta_{21},\phi) = \tan^{-1}\left[\frac{1}{\zeta_{21}}\tan\phi(t)\right] \quad (8)$$

$$\zeta_{21} = \frac{J_2(\Delta\phi_m)}{J_1(\Delta\phi_m)}. \quad (9)$$

Using a PLL locked at the centre frequency ω_c in order

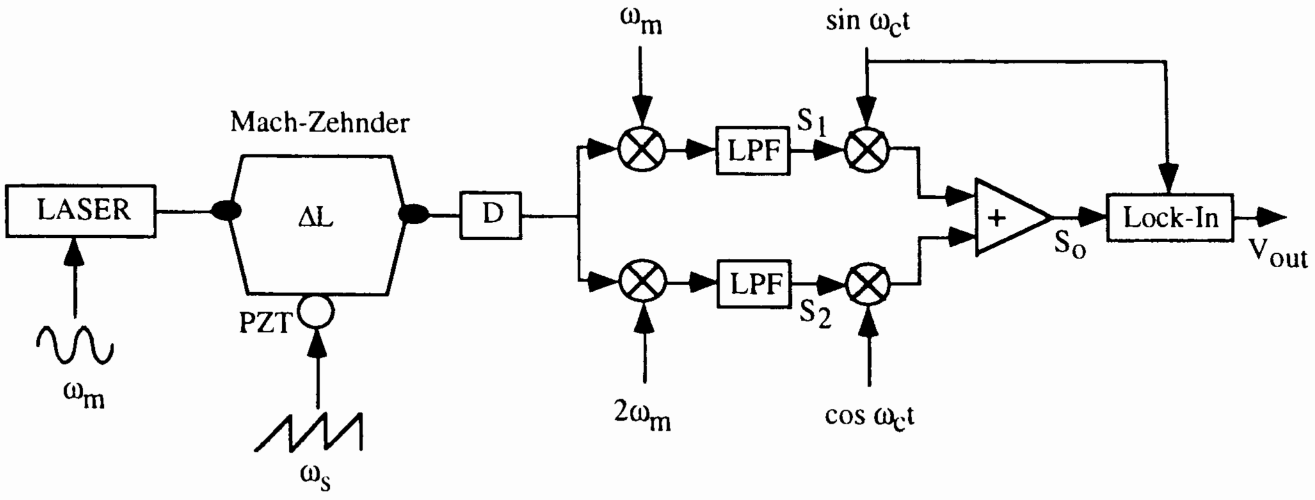


Figure 1. **Schematic diagram of the synthetic-heterodyne demodulation scheme (LPF: low pass filter; ΔL: path imbalance of the interferometer).**

to process the signal (6) the resulting output signal, $V_{out}(J_1 \neq J_2)$, is given by

$$V_{out}|_{J_1 \neq J_2} = \alpha I_0 k J_1(\Delta\phi_m) \frac{\partial\phi(t)}{\partial t} H(\zeta_{21}, \phi)$$
$$= V_{out}|_{J_1=J_2} H(\zeta_{21}, \phi), \quad (10)$$

where

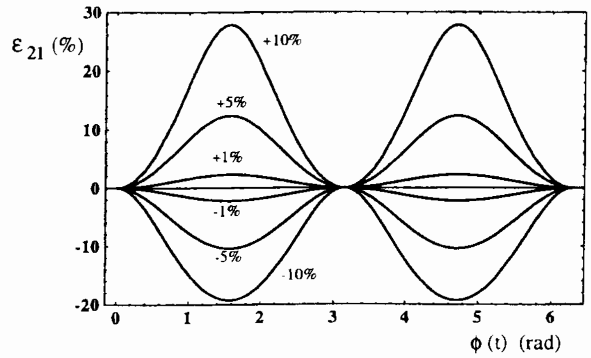
$$H(\zeta_{21}, \phi) = F(\zeta_{21}, \phi) \frac{1}{\zeta_{21}\cos^2\phi(t) + \frac{1}{\zeta_{21}}\sin^2\phi(t)} \quad (11)$$

when $J_1 = J_2$, $H(\zeta_{21}, \phi) = 1$ and the free of error demodulated phase signal is obtained.

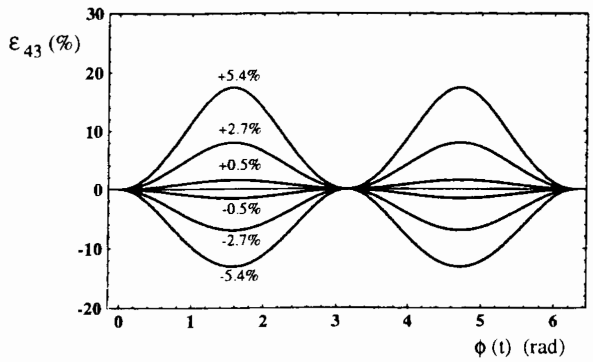
It is important to notice that $F(\zeta_{21}, \phi)$ and $H(\zeta_{21}, \phi)$ depend on the quasi-static phase $\phi_d(t)$, which changes randomly and, therefore, variable calibration errors will be introduced in the demodulated phase signal.

To evaluate this effect, we define the error function, ϵ_{21}, as the relative deviation of $V_{out(J1 \neq J2)}$ from the ideal case $V_{out(J1=J2)}$, giving

$$\epsilon_{21}(\%) = \left[\frac{V_{out}|_{J_1 \neq J_2} - V_{out}|_{J_1=J_2}}{V_{out}|_{J_1=J_2}}\right] \times 100\%$$
$$= [H(\zeta_{21}, \phi) - 1] \times 100\%. \quad (12)$$



(*a*)



(*b*)

Figure 2. **Error functions $\epsilon_{i+1,i}(i = 1, 3)$ as a function of the phase $\phi(t)$ for deviations in $\Delta\phi_m$ from the optimum values $\Delta\phi_{m0i}$ of $\pm 0{\cdot}01\Delta\phi_{m01}$, $\pm 0{\cdot}05\Delta\phi_{m01}$ and $\pm 0{\cdot}1\Delta\phi_{m01}$: (*a*) Bessel pair $\{J_1, J_2\}$; (*b*) Bessel pair $\{J_3, J_4\}$.**

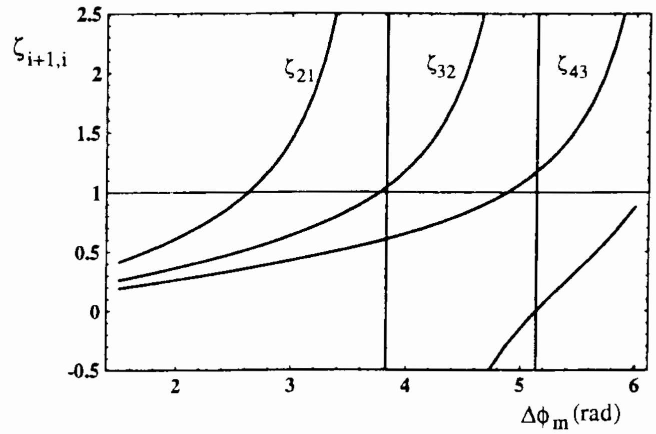


Figure 3. **Ratios $\zeta_{i+1,i}(i = 1, 2, 3)$ as a function of the amplitude modulation $\Delta\phi_m$.**

Figure 2(*a*) shows this error function (12) versus the phase $\phi(t)$ for deviations in $\Delta\phi_m$ of $\pm 0{\cdot}01\Delta\phi_{m01}$, $\pm 0{\cdot}05\Delta\phi_{m01}$ and $\pm 0{\cdot}1\Delta\phi_{m01}$ from the optimum value $\Delta\phi_{m01}$, which correspond to relative deviations of $\pm 1\%$, $\pm 5\%$ and $\pm 10\%$, respectively. This optimum value, where the error is zero ($J_1 = J_2$, i.e. $\zeta_{21} = 1$) is $\Delta\phi_{m01} = 2{\cdot}63$ rad. It is interesting to notice that the error functions associated with relative deviations of $+1\%$ and -1% are similar; however, for the $\pm 10\%$ case, these functions differ by as much as $\approx 40\%$. This effect is due to the ratio ζ_{21}, which is highly asymmetric with respect to the optimum value $\Delta\phi_{m01}$, as can be seen from figure 3.

This figure also shows other high-order ζ ratios. It is evident that for identical deviations from the optimum values $\Delta\phi_{m0i}$ ($i = 1, 2, \ldots$), the parameter $\zeta_{i+1,i}$ decreases relative to the standard case, $i = 1$. This means that, if instead of the Bessel pair $\{J_1, J_2\}$ a high-order Bessel pair is used, such as $\{J_i, J_{i+1}\}$, with $i > 1$, the result would be more favourable, being the output signal given by

$$V_{out}|_{J_i \neq J_{i+1}} = V_{out}|_{J_i=J_{i+1}} H(\zeta_{i+1,i}, \phi), \quad (13)$$

where the variables involved in this equation are identical to those defined in relations (7), (8) and (9), only substituting the pair $\{J_1, J_2\}$ by the pair $\{J_i, J_{i+1}\}$. As an example, figure 2(*b*) shows the relative error, $\epsilon_{43}(\%)$, corresponding to the Bessel pair $\{J_3, J_4\}$, as a function of the phase $\phi(t)$ for deviations in $\Delta\phi_m$ from the optimum value $\Delta\phi_{m03}$ (= 4·88 rad when $J_3 = J_4$, i.e. $\zeta_{43} = 1$) of $\pm 0{\cdot}01\Delta\phi_{m01}$, $\pm 0{\cdot}05\Delta\phi_{m01}$ and $\pm 0{\cdot}1\Delta\phi_{m01}$, which are the same as the ones considered in figure 2(*a*) (these deviations correspond to relative deviations with respect to $\Delta\phi_{mo3}$ of ±0·54%, ±2·7% and ±5·4%, respectively). Comparing these two error functions (figures 2(*a*) and 2(*b*)), a deviation of $0{\cdot}01\Delta\phi_{m01}$ for the case $\{J_3, J_4\}$ gives a phase demodulated error ≈35% smaller when compared with the solution $\{J_1, J_2\}$; and for a deviation of $0{\cdot}1\Delta\phi_{m01}$, the improvement is ≈40%. These two samples illustrate the advantages of processing high-order Bessel pairs $\{J_1, J_{i+1}\}$, with $i > 1$.

3. Experiment and results

To evaluate the function $F(\zeta_{21}, \phi)$ (equation (7)) a Mach-Zehnder interferometer with a path imbalance of ≈3·83 cm was built using standard single-mode optical fibre. The optical source was a semiconductor laser (Melles-Griot 06DLS407, $\lambda = 785$ nm, temperature controlled), operating 20 mA above threshold, and with a measured effective frequency-to-current factor of 5·4 GHz mA^{-1}. The injection current of the laser was modulated by a sinewave with a frequency of 24 kHz and amplitude of Δi_m ($\Delta i_{m01} = 0{\cdot}42$ mA for the case $\Delta\phi_{m01} = 2{\cdot}62$ rad, i.e. $J_1 = J_2$). A PZT stretcher was mounted in one arm of the interferometer to induce the phase signal information $\phi_s(t)$. This PZT had an efficiency of 0·37 rad V^{-1} and was driven by a sawtooth waveform (frequency =30 Hz, amplitude = 17 V) to sweep the interferometer over one fringe. The electronic carrier frequency ($\omega_c/2\pi$) used in the heterodyne circuit was 10 kHz. Figures 4(*a*) and 4(*b*) show the output signal (S_0) as a function of time (which is proportional to $\phi(t)$ considering that $\phi(t) \approx 2\pi f_{PZT} t$ during the ramp period) for 1% and 25% relative deviations of ζ from one, which was achieved by changing the optimum amplitude value of the modulation current (optimum value: $\Delta i_{m01} = 0{\cdot}42$ mA). For the same deviations, figure 5 shows the theoretical error amplitude modulation $A(\zeta_{21}, \phi) = J_1(\Delta\phi_m) F(\zeta_{21}, \phi)$.

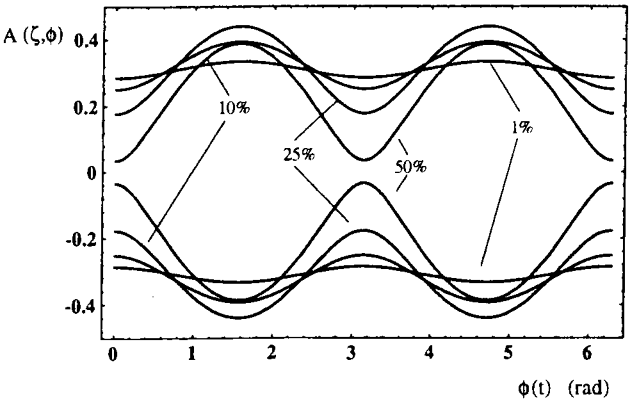


Figure 5. Theoretical error amplitude modulation $A(\zeta, \phi) = J_1(\Delta\phi_m)F(\zeta, \phi)$ versus phase $\phi(t)$ for different relative deviations of ζ_{21} from one.

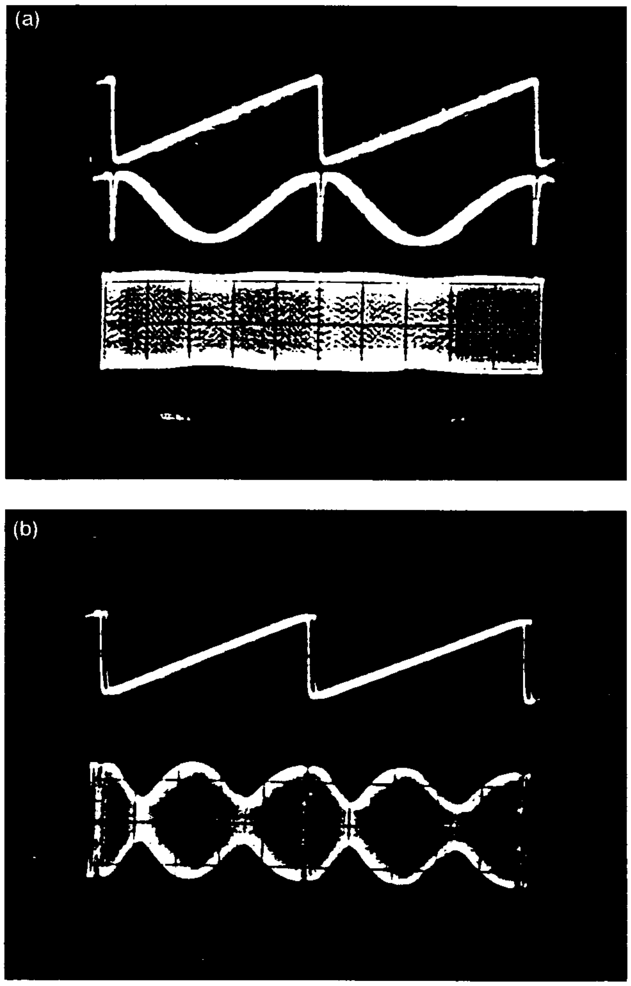


Figure 4. Demodulated output signal S_0 (bottom trace) when the PZT is driven by a sawtooth waveform (top trace: amplitude = 17 V, frequency = 30 Hz) to sweep the interferometer over one fringe (figure 4(*a*)—middle trace) for: (*a*) 1% and (*b*) 25% relative deviation of ζ_{21} from one.

Comparing the experimental results of figure 4 with the corresponding theoretical ones given in figure 5 the global agreement is evident. Thus, the results (7) and (8) (and therefore, the result (11)) can be used to quantify the readout phase error generated by this synthetic-heterodyne processing scheme and also to evaluate the fluctuations of the small signal sensitivity. As mentioned before, for a single interferometric sensor this (variable) error can be virtually eliminated through proper adjustment of the laser injection current amplitude modulation. However, in an array of interferometric sensors powered by a single optoelectronic unit these errors can not, in general, be avoided, because it is not possible to perform the total equilization of the path imbalances of all the sensors. As was demonstrated by figures 2(*a*) and 2(*b*), if instead of the Bessel pair $\{J_1, J_2\}$—equations

(3.1), (3.2)—high-order Bessel pairs (such as $\{J_3, J_4\}$) are used, these readout errors can be successively decreased. Normally, the utilization of high-order Bessel pairs will require larger path-imbalances for the interferometric sensors with the corresponding increase in the system noise level due to the increase of phase noise. This effect can be negligible or not depending on the relative contributions of the system noise sources. In general, it is reasonable to state that, in a practical situation, a compromise is required between the susceptibility of the system to readout phase errors and the allowable level of noise.

In conclusion, the synthetic-heterodyne signal processing technique, frequently used to demodulate the phase information of interferometric fibre-optic sensors, was analysed from the viewpoint of the phase readout errors generated when there is a mismatch from the ideal conditions. These errors were quantified and experimentally tested, and their implications on the performance of arrays of interferometric sensors demodulated using this technique were assessed.

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